Written methods for multiplication of whole numbers

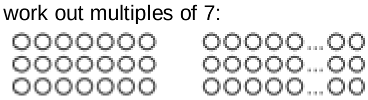
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10
- partition number into multiples of one hundred, ten and one
- work out products such as 70 × 5, 70 × 50, 700 × 5 or700 × 50 using the related fact 7 × 5 and their knowledge of place value
- add two or more single-digit numbers mentally
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Method	Example
Stage 1: Mental multiplication using partitioning	
Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.	Informal recording in Year 4 might be: 43 40 + 3 $\downarrow$ $\downarrow$ × 6 240 + 18 = 258 Also record mental multiplication using partitioning: $14 \times 3 = (10 + 4) \times 3$ $= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6$ $= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$ Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7: 

<table>
<tr><th>Method</th><th>Example</th></tr>
<tr><td></td><td>$7 \times 3 = (5+2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$</td></tr>
<tr><td colspan="2">Stage 2: The grid method</td></tr>
<tr><td> As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps. It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products. </td><td> 38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266
$$\begin{array}{c|r} \times & 7 \\ \hline 30 & 210 \\ \hline 8 & 56 \\ \hline & 266 \end{array}$$</td></tr>
<tr><td> The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56. The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4. </td><td>
$$\begin{array}{c|r} & 30 + 8 \\ \hline \times & 7 \\ \hline & 210 \\ \hline & 56 \\ \hline & 266 \end{array}$$</td></tr>
<tr><td colspan="2">Stage 3: Expanded short multiplication</td></tr>
<tr><td> The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above. Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed. Most children should be able to use this expanded method for TU × U by the end of Year 4. </td><td>
$$\begin{array}{r} 30+8 \\ \times\ \underline{\quad 7} \\ 210 \\ \underline{56} \\ \underline{\underline{266}} \end{array} \qquad \begin{array}{r} \\ \\ 30 \times 7 = 210 \\ 8 \times 7 = \ \ 56 \\ \\ \end{array} \qquad \begin{array}{r} 38 \\ \times\ \underline{\quad 7} \\ 210 \\ \underline{56} \\ \underline{\underline{266}} \end{array}$$</td></tr>
<tr><td colspan="2">Stage 4: Short multiplication</td></tr>
</table>

<table>
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<tr><td>
The recording is reduced further, with carry digits recorded below the line.

If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.
</td><td>
38
× 7
266
5
The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.
</td></tr>
<tr><td colspan="2">Stage 5: Two-digit by two-digit products</td></tr>
<tr><td>
Extend to TU × TU, asking children to estimate first.

Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.
</td><td>
56 × 27 is approximately 60 × 30 = 1800.
<table>
<tr><td>×</td><td>20</td><td>7</td><td></td></tr>
<tr><td>50</td><td>1000</td><td>350</td><td>1350</td></tr>
<tr><td>6</td><td>120</td><td>42</td><td>162</td></tr>
<tr><td></td><td></td><td></td><td>1512</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table>
<table>
<tr><td></td><td>50</td><td>6</td><td></td></tr>
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<tr><td></td><td></td><td></td><td>1</td></tr>
</table>
</td></tr>
<tr><td>
Reduce the recording, showing the links to the grid method above.
</td><td>
56 × 27 is approximately 60 × 30 = 1800.
56
× 27
1000   50 × 20 = 1000
120   6 × 20 = 120
350   50 × 7 = 350
42   6 × 7 = 42
1512
1
</td></tr>
<tr><td>
Reduce the recording further.

The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally.

The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.
</td><td>
56 × 27 is approximately 60 × 30 = 1800.
56
× 27
1120   56 × 20
392   56 × 7
1512
1
</td></tr>
<tr><td colspan="2">Stage 6: Three-digit by two-digit products</td></tr>
<tr><td>
Extend to HTU × TU asking children to estimate first. Start with the grid method.

It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.
</td><td>
286 × 29 is approximately 300 × 30 = 9000.
</td></tr>
</table>

<table>
<tr><th>Method</th><th>Example</th></tr>
<tr><td></td><td>
<table>
<tr><td>×</td><td>20</td><td>9</td><td></td></tr>
<tr><td>200</td><td>4000</td><td>1800</td><td>5800</td></tr>
<tr><td>80</td><td>1600</td><td>720</td><td>2320</td></tr>
<tr><td>6</td><td>120</td><td>54</td><td>174</td></tr>
<tr><td></td><td></td><td></td><td>8294</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table>
</td></tr>
<tr><td>Reduce the recording, showing the links to the grid method above.

This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.</td><td>
<table>
<tr><td>286</td><td></td></tr>
<tr><td>× 29</td><td></td></tr>
<tr><td>4000</td><td>200 × 20 = 4000</td></tr>
<tr><td>1600</td><td>80 × 20 = 1600</td></tr>
<tr><td>120</td><td>6 × 20 = 120</td></tr>
<tr><td>1800</td><td>200 × 9 = 1800</td></tr>
<tr><td>720</td><td>80 × 9 = 720</td></tr>
<tr><td>54</td><td>6 × 9 = 54</td></tr>
<tr><td>8294</td><td></td></tr>
<tr><td>1</td><td></td></tr>
</table>
</td></tr>
<tr><td>Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.

Again, the carry digits in the partial products are usually carried mentally.</td><td>286 × 29 is approximately 300 × 30 = 9000.
<table>
<tr><td>286</td><td></td></tr>
<tr><td>× 29</td><td></td></tr>
<tr><td>5720</td><td>286 × 20</td></tr>
<tr><td>2574</td><td>286 × 9</td></tr>
<tr><td>8294</td><td></td></tr>
<tr><td>1</td><td></td></tr>
</table>
</td></tr>
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